Zeeman effect

Prob1: A spectrometer can just resolve a wavelength 0.5 Å at 5000 Å. What should be the strength of external magnetic field to observe the normal Zeeman effect

Sol: The Zeeman shift is $\Delta\nu = \frac{eB}{4\pi m}$.

we have, $\nu = \frac{c}{\lambda} \Rightarrow \Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$

$\therefore \Delta\lambda = \frac{\lambda^2}{c}\frac{eB}{4\pi m}$ or $B = \frac{\Delta\lambda}{\lambda^2} c \frac{4\pi m}{e}$

we have $\Delta\lambda = 0.5$ Å and $\lambda = 5000$ Å, $B \sim 4.29$ wb/m^2.

Prob2: One of the most prominent spectral lines of Calcium is one with wavelength $\lambda = 4226.73$ Å. Calcium atoms exhibit normal Zeeman patterns when placed in a uniform magnetic field of 4 wb/m^2. Calculate the wavelength of the three components of normal Zeeman pattern & the separation between them.

Soln: The Zeeman separation is $\Delta\nu = \frac{eB}{4\pi m}$

& $\Delta\lambda = \frac{\lambda^2}{c}\frac{eB}{4\pi m} \sim 0.33$ Å

$\therefore$ The wavelengths of the normal Zeeman components are (4226.73 ± 0.33) Å.

3. Find the equatorial velocity v of an electron under the assumption that it is a uniform sphere of radius $r = 5.00 \times 10^{-13}$ that is rotating about an axis through its centre.

Soln: The angular momn is $L = I\omega = \frac{2}{5} mr^2\omega$ & $\omega = \frac{v}{r}$ is its angular velocity. The total spin angular momentum is

$S = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar = \frac{2}{5} mr^2\frac{v}{r} = \frac{2}{5} mvr$

$\Rightarrow v = \frac{5\sqrt{3}}{4}\frac{\hbar}{mr} \sim 5.01 \times 10^{12}$ ms^{-1} $\sim 10^4 c \sim$ impossible

Prob: Calculate the possible orientation of spin vector $\vec{S}$ with respect to magnetic field direction.

Sol: $\cos\theta = \frac{S_z}{S} = \frac{m_s \hbar}{\sqrt{s(s+1)}\,\hbar} = \frac{m_s}{\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}} = \frac{2}{\sqrt{3}} m_s$

now, $m_s = \pm \frac{1}{2}$ so $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$= 54.7°$ and $125.3°$.

Prob: A beam of electrons enters a uniform magnetic field of flux density $1.2\ Wb/m^2$. Find the energy difference in eV between electrons whose spins are parallel & anti parallel to the field.

Soln: $\vec{\mu_s} = -\frac{e}{m}\vec{S}$, Along z axis, $\mu_{sz} = -\frac{e}{m} S_z = \mp \frac{eh}{4\pi m}$

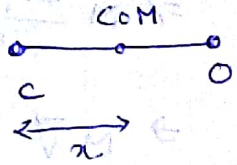
$\therefore U_m = -\vec{\mu_s}\cdot\vec{B} = -\mu_{s_z} B = \mp \frac{ehB}{4\pi m}$

$\therefore$ difference $= \frac{ehB}{4\pi m} - \left(-\frac{ehB}{4\pi m}\right) = \frac{ehB}{2\pi m}$.

Centre of mass problems.

→ The distance between the centre of Carbon and oxygen atoms in the carbon monoxide (CO) gas molecule is 1.130×10^{-10} m. Locate the centre of mass of the molecules relative to the carbon atom.

Soln: The CO gas molecules is a linear molecule.



So the COM is likely to remain in the line joining the C carbon and Oxygen atom. So if x is the distance of COM from C, then we can always write

$\Rightarrow 12x = 16(1.130\times10^{-10} - x) \Rightarrow 28x = 16\times1.130\times10^{-10}$

$\Rightarrow x = 6.457\times10^{-10}$ m.

Thus, the splitting is shown below, with the selection rules

$\Delta M_J = 0, \pm 1$ ($M_J = 0 \not\leftrightarrow M_J = 0$ if $\Delta J = 0$).

→ The Paschen - Back Effect :-

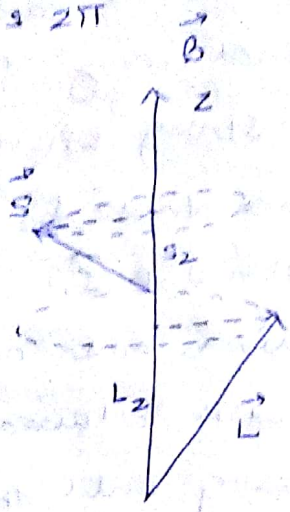
In anomalous Zeeman effect, remember that the applied external magnetic field was weak. This means that the internal mag. field produced due to the motion of $\bar{e}$ is ~~less~~ strong enough as compared to the external B. This means that for such an atom, $\vec{L}$ and $\vec{S}$ keeps on precessing around $\vec{J}$.

However, when one increases the mag. field $\vec{B}$ (external), the separation between the anomalous Zeeman components keep on increasing until they exceed the separation between multiplet fine-structure components. (For eg- in case of D_1 & D_2 lines in a weak mag. field (i.e. anomalous Zeeman effect) if you keep on increasing the external magnetic field $\vec{B}$, then at one point of time, the separation between the levels will keep on increasing in such a way that at one point of time there will be overlapping between the levels (see the various M_J levels in anomalous Zeeman effect in page 208, Rajkumar). As a result at some point of time instead of 4 & 6 lines from D_1 and D_2, we shall see only three lines exactly similar to the normal Zeeman effect). This is the Paschen Back effect.

Explanation :- As mentioned previously, when a mag. field $\vec{B}$ is applied & becomes stronger than the internal fields, $\vec{J}$ starts precessing around $\vec{B}$ very fast. Remember that in a weak mag. field, the precession of $\vec{L}$ & $\vec{S}$ is fast around $\vec{J}$ as compared to $\vec{J}$ around $\vec{B}$. Thus, the coupling between $\vec{L}$ & $\vec{S}$ remains intact. On applying a strong B field, the exactly

reverse scenario happens. $\vec{S}$ starts precessing around $\vec{B}$ very fast as compared to $\vec{L}$ & $\vec{S}$ around $\vec{J}$. As the field is increased more, $\vec{L}$ & $\vec{S}$ starts independently precessing around $\vec{B}$. While precessing independently, the components of $\vec{L}$ & $\vec{S}$ around $\vec{B}$ gets quantised so that

$$L_z = M_L \frac{h}{2\pi} \quad \& \quad S_z = M_s \frac{h}{2\pi}$$



where $M_L = L, L-1, \ldots, -L$ &

$M_s = S, S-1, \ldots, -S$.

The velocity of these independent precessions are

$$\omega_L = \frac{e}{2m} B \quad \& \quad \omega_S = 2 \frac{e}{2m} B$$

& The interacting energy of these precession are

$$\Delta E_L = \omega_L L_z = \frac{e}{2m} B M_L \hbar \quad \text{(Refer previous class)}$$

$$\Delta E_S = \omega_S S_z = 2 \frac{e}{2m} B M_s \hbar$$

∴ the total interaction energy is

$$\Delta E = (M_L + 2M_S) \frac{eh}{4\pi m} B$$

& wave number shift is,

$$-\Delta T = \frac{\Delta E}{hc} = (M_L + 2M_S) \frac{eB}{4\pi mc}$$

In Lorentz unit,

$$-\Delta T = (M_L + 2M_S) L'.$$

This is the expression for strong-field interaction energy. Now M_L has $(2L+1)$ values & M_s has $(2S+1)$ components. So each of the field free level is split into $(2L+1)(2S+1)$ levels.

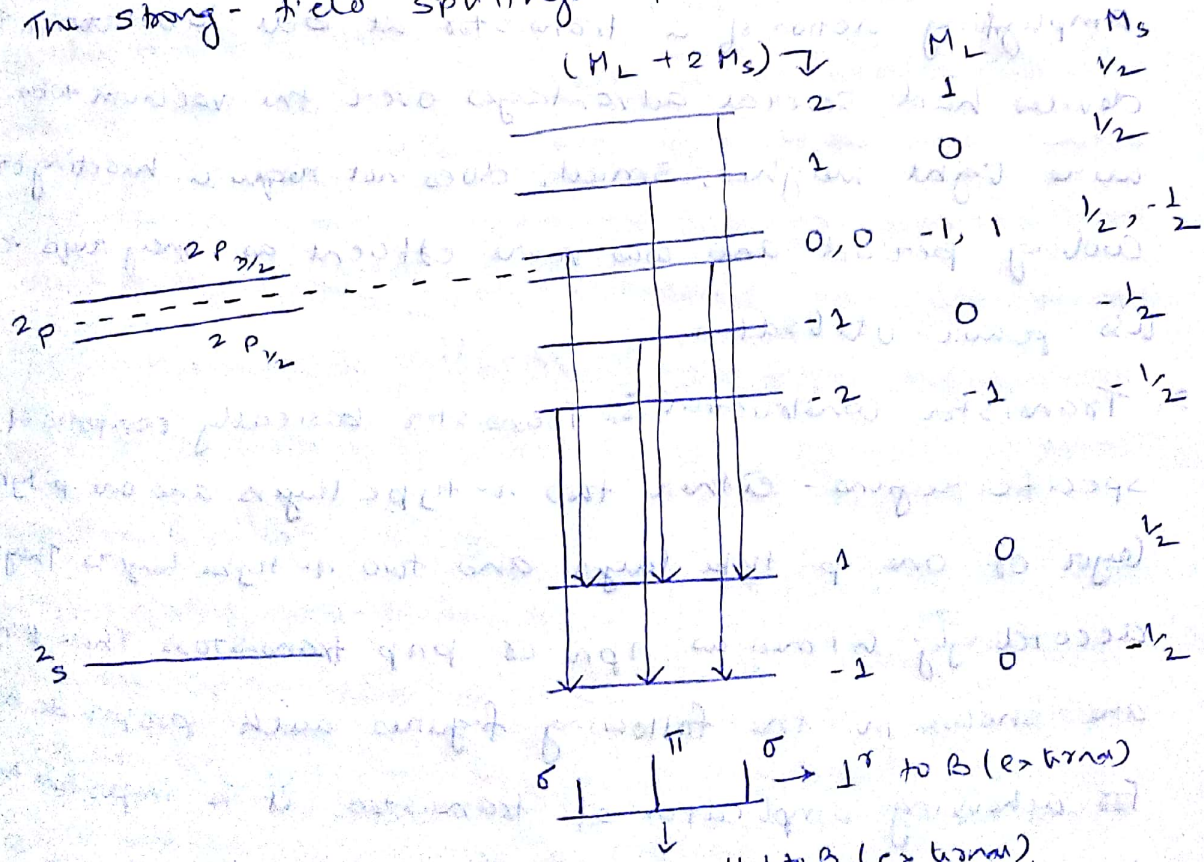
Let us consider the following examples

$^2P - {}^2S$

This is the transition responsible for D_1 and D_2. Now,

Term	No. of Strong-field levels $(2L+1)(2S+1)$	M_L	M_s	Shift in L unit $(M_L + 2M_s)$
^2P $L=1, S=\frac{1}{2}$	6	1	$\frac{1}{2}, -\frac{1}{2}$	2, 0
		0	$\frac{1}{2}, -\frac{1}{2}$	1, -1
		-1	$\frac{1}{2}, -\frac{1}{2}$	0, -2
^2S $L=0, S=\frac{1}{2}$	2	0	$\frac{1}{2}, -\frac{1}{2}$	1, -1

The strong-field splittings of the terms ^2P & ^2S have



$\Delta M_L = 0$ ($\parallel$ to field)

$\Delta M_L = \pm 1$ ($\perp^r$ to field)

$\Delta M_s = 0$.